The Taxation of Digital Services as a Rent-Extracting Policy[1]

by

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Abstract:

The use of digital services is largely non-rival. Their supply is only profitable if some form of market power is exercised. A monopolized supply incentivizes importing countries to pursue rent-extracting policies. Small countries are particularly incentivized to tax the import of digital services which contrasts with well-known findings in trade theory. Indeed, various countries have already introduced special taxes on digital services. If such practice spreads, the quality of digital services will be negatively affected. This paper argues that countries exporting digital services have good reason to respond by promoting an international tax regime in which the profit earned on the remote supply of digital business services is split between the countries involved. Such profit splitting is similarly simple, yet different from the profit allocation rule that the Secretariat of the OECD (2020) uses to illustrate its proposed consensus solution to the tax challenges arising from digitalization.

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1. Introduction

The digitalization of the economy presents serious challenges for international corporate income taxation (OECD, 2015; Commission Expert Group, 2014). One such challenge is raised by the cross-border supply of remote digital services. Such services are supplied without relying on a permanent establishment in the customer's country of residence. In the conventional view, this is business without nexus in the destination country. Current international tax rules assign the right to tax the profit earned on remote supplies to the seller's country of residence.

This assignment of taxing rights is increasingly being challenged in its application to digital services. As a matter of fact, quite a number of countries have implemented taxes on digital services (DSTs), such as Austria, Hungary, India, Indonesia, Italy, Tunisia, Turkey, and the United Kingdom. Other countries, such as Pakistan, Thailand, Turkey, and Uruguay, have decided to impose gross-based withholding taxes on digital businesses. Still other countries use diluted requirements for permanence and physical presence to establish nexus for net-basis taxation. (For details see Bunn et al., 2020; OECD, 2018, Chap. 4.) In what follows, the term "digital tax" is used for all such provisions designed to expand source taxation of online business activities.

This paper explains the increasing spread of digital taxes by the attempt of importing countries to extract suppliers' rent income. The analysis extends that of Brander and Spencer (1984) by considering goods and services which are not only supplied in variable quantity but also in variable quality. The development of quality is costly whereas the production of quantity is not necessarily. Rather, it is a characteristic of digital services that the (variable) cost of servicing additional customers is negligible (Commission Expert Group, 2014). Without the exercise of market power the supply would not pay off. The model assumes a monopolized remote supply which provides the incentive for importing countries to pursue a rent-extracting policy.

It is shown that the incentive of a market country to tax the import of monopolized supplies is considerably strengthened if the import concerns digital services and if the market share of this country is sufficiently small. Such a market country is even incentivized to skim off the full profit contribution net of tax earned by the monopolist in that country. There is no cost of internal efficiency. The reason is that the tariff has a vanishing effect on supplied quality if the importing country is sufficiently small. Such a theoretical result may well explain why so many small countries have introduced digital taxes. An important precondition, however, is that the monopolist sets country-specific prices. A prime example of this in e-commerce is Google Ads, where price discrimination by country results from the automated auctioning of advertising space.[2] There may be reasons why country-specific prices are neither feasible nor optimal. An example is Amazon. The commission rates charged by Amazon on business selling in its online marketplaces are known to differ widely between product categories but hardly at all between marketplaces. A policy of rent extraction, however, is self-defeating if the monopolist does not set country-specific prices.[3]

If the mass of market countries succeeds in skimming off the rent income earned on service imports, a drastically inefficient equilibrium results. The monopolist may be unable to earn the profit contributions on exports needed to cover the cost of developing quality. In extreme cases, the digital service is not provided at all and the digital tax does not generate any revenue. The monopolist's home country has only limited possibilities to take unilateral action against this. A policy of retaliation is not as credible as it would be in the old economy. This is so as trade in digital services tends to lack reciprocity. For technological reasons, small countries suffer a comparative disadvantage in commercializing digital services. This biases their stance on trade policy. Retaliation would have to be targeted at a non-digital sector, which could be considered a disproportionate action and initiate an escalating trade war.

In Section 10 I argue that an appealing solution is an internationally coordinated tax regime in which the profit earned on remote digital business services is split for the purpose of taxation. Such profit splitting can be expected to result when the monopolist's home country bargains with market countries over the allocation of taxable profit. High-tax countries with a strong digital sector even have reason to endorse profit splitting as it provides certain resilience against tax competition for the location chosen for developing digital services.

The paper is structured as follows: After Section 2 has briefly summarized the relevant literature Sections 3 and 4 introduce a simple model which allows analyzing the remote supply of digital as well as non-digital services and goods. Both forms are discussed in parallel in order to work out

[2] For an introduction to Google Ads pricing see https://www.webfx.com/blog/marketing/much-cost-advertise-google-adwords/

[3] Taking this effect into account provides a strong argument in favor of the narrow based Austrian DST which only taxes the revenues from online advertising and it should be an argument against the broad based French and British DSTs which include revenues earned by Amazon on its online marketplaces (Bunn et al., 2020).

the differential effects a tax on imports has for the new and the old economy. In Sections 5 to 7, the focus is on a market country and its incentives to pursue a rent-extracting policy. The main insight is due to Brander and Spencer (1984). Section 7 extends their analysis to the case of endogenous quality. Section 8 studies the case where the market countries join to pursue a rent-extracting policy. Section 9 discusses policy options of the monopolist's home country. Section 10 makes a case for international policy coordination. It is argued that a country exporting digital business services has good reason to negotiate over internationally coordinated profit splitting. Section 11 concludes.

2. Related literature

This paper interprets the taxation of digital services as a rent-extracting policy. The key reference is Brander and Spencer (1984). Their analysis is extended by differentiating between the quantity and quality of imports. The modelled characteristic of digital services is a lack of rivalry in their use, the marginal cost of quantity being zero. As the marginal cost of quality is yet assumed to be positive and increasing, the supply of digital services is only profitable if some form of market power is exercised. Given this framework, a tax on digital services is featured as an instrument designed to extract rent income earned by service suppliers. This is similar in spirit to Cui and Hashimzade (2019) who stress an analogy between DSTs and resource royalties.

The OECD (2018) provides an in-depth analysis of the common characteristics of digital businesses. Three characteristics are emphasized: cross-jurisdictional scale without mass, heavy reliance on intangible assets, and the importance of data, user participation and their synergies with intangible assets. Low marginal cost of supply is repeatedly mentioned in the text but it is not among the characteristics highlighted. The present paper considers the insignificance of the marginal cost of quantity as crucial for understanding tax policy incentives in the digital economy.

Google Ads will figure as a prime example for this paper's analysis. There is a strand of literature stressing the two-sidedness of Google and other internet platforms such as Amazon and Facebook (Kind et al., 2008; Cui et al., 2019; Koethenbuerger, 2020; Bloch et al., 2021). In the present paper, two-sidedness is not explicitly modeled. It is however captured in reduced form by the quality dimension of the digital service under consideration. See Section 3.

The following analysis focuses on the corporate income taxation of digital business services. Services provided from business to consumers (B2C) are left aside. Their taxation raises implementation and enforcement issues that go beyond the scope of this paper. See rather, among others, Hellerstein (2016), Agrawal et al. (2017), and OECD (2018). Note also that the bulk of e-commerce is B2B sales (Báez et al., 2020). According to UNCDAT (2020), it accounted for 83% of almost $26 trillion that e-commerce sales reached in 2018.

The role of digital taxes as part of international corporate taxation is judged controversially in the literature. Some authors meet such taxes with outright rejection. E.g., Olbert et al. (2019) see “no justification for introducing a new tax order for digital businesses”. With reference to the European Commission’s (2018) proposed DST they argue that it is “clearly ring-fencing, bears a substantial risk of double taxation and legal uncertainty, and most likely, does not justify its administrative costs”. On the other hand, there is equally consensus, that the “current rules of the international tax system are unfit for the digital age” (Becker et al., 2018). The European Commission (2018) justifies its proposal for a DST with the widespread concern that profits earned in the digital economy are not effectively and fairly taxed. Above all, it is increasingly difficult to turn a blind eye to the fact that digital taxes are on the rise worldwide.

The OECD and G20 countries have responded to the development and invited other countries to jointly work out a consensual solution within the Inclusive Framework on BEPS. In 2019, the Secretariat of the OECD has presented a proposal (“Unified Approach”) to facilitate a consensus solution on Pillar One of the Programme of Work adopted by the Inclusive Framework in May 2019. The Secretariat’s approach is based on a new concept of nexus which is not dependent on physical presence and on a new profit allocation rule which goes beyond the Arm’s Length Principle (OECD, 2019). For the present paper, the latter is more relevant than the former. The Secretariat proposes a scheme based on separating a multinational enterprise’s (MNE) total profit into a routine and a residual part. A share of deemed residual profit would be allocated to market countries using a formula based on sales. Such a proposal belongs to a family of similar proposals, which are referred to as Residual Profit Allocation. They may well differ in details, such as the factor(s) used for apportioning residual profit across countries. E.g., Devereux et al.

(2019) would base the apportionment on the location of residual gross income, rather than sales. By contrast, the European Commission (2011, 2015, 2018) promotes a scheme in which the total, rather than just the residual, profit of an MNE is apportioned between jurisdictions according to a formula weighting labor, capital (assets), and sales as indicators of the MNE's local activity. (See De Mooij et al., 2019, for a recent assessment of global formula apportionment.) The OECD Secretariat's proposal is notable for its simplicity. In an impact analysis prepared to illustrate the proposal, it is simply assumed that 20 percent of taxable residual profit is allocated to market jurisdictions (OECD, 2020). Simplicity is also a characteristic feature of the proposal of Báez et al. (2020) to introduce a "withholding tax on all base-eroding payments to non-residents, with specific standard exemptions from such withholding tax for payments made to payees registered to be domestically taxed under the normal net taxation scheme". If the design of such a withholding tax is internationally coordinated it is not too far from profit allocation by profit splitting which is the solution for which the present paper provides positive-theoretic arguments. A normative justification of profit splitting is provided by Richter (2019) by elaborating on the OECD's objective to align profit taxation with value creation (OECD, 2015).

3. A model of remote supplies with country-specific price discrimination

The focus is on a monopolist supplying foreign markets. In the base model, the monopolist is assumed not to sell to its own country labeled H for Home. The sales rather go to $N+1$ foreign market countries M_i $(i=0,..,N)$. They are remote sales from H and are made without relying on a permanent establishment in M_i. Country M_0 is special in the sense that it plans to extract a share of the supplier's monopoly rent. All other countries M_i $(i=1,..,N)$ are assumed to stay passively. Their sole role is to feature the existence of third markets.

The product is sold in quantity and quality. The quantity sold in M_0 is denoted by x and that in M_i by X_i for $i=1,..,N$. The marginal cost of quantity is constant and non-negative, $W \geq 0$. If the marginal cost is positive, $W>0$, the use of quantity is rival and if $W=0$, the use is non-rival. Quality is denoted by Q. The cost of producing quality, $C(Q)$, is positive, increasing, and convex. The provision of digital services is modeled by $W=0$ and the provision of non-digital goods and services is captured by $W>0$. According to this definition, a computer is a non-digital good, as the production of hardware is costly even at the margin. $p(x,Q)$ is average revenue earned in M_0 and $P^i(X_i,Q)$ that earned in M_i $(i=1,..,N)$. Inverse demand functions of M_i are identical, $P^i \equiv P$, and revenue functions, $xp(x,Q)$, $X_iP(X_i,Q)$ concave by assumption.

Profits are taxed. Let T and t be the tax rates applied in H and M_0, respectively. If current law applied, revenue from M_0, $xp(x,Q)$, would be taxed as seller's revenue in H. In contrast, buyers in M_0 would be allowed to deduct their expenses from own taxable revenue. As a result, $xp(x,Q)$ would be taxed only once, and profit taxation would be non-distortionary even if the tax rates were not the same, $T \neq t$. However, the base model is to reflect the increasing tendency of countries to break these rules. Thus it is assumed that payments from M_0 to H are taxed both in M_0 and in H, so tending to make taxation distortionary. H may feel justified in taxing $xp(x,Q)$ as these payments are foreign revenues earned by a resident firm. Without policy coordination, M_0 may feel equally justified in disallowing the deduction of $xp(x,Q)$ as these expenses remain otherwise untaxed in M_0. From the joint perspective of both countries, $xp(x,Q)$ is a (pure) profit contribution if $W=0$ and it is revenue with allocable cost, Wx, if $W>0$.

The monopolist is assumed to maximize profit after tax, Π^a. The behavioral implications are the same as if the monopolist maximizes $\Pi(X_i,x,Q;\theta)=\Pi^a/(1-T)$ with

$$\Pi \equiv \sum_{i=1}^{N}[X_iP(X_i,Q)-WX_i]+(1-\theta)xp(x,Q)-Wx-C(Q) \qquad (1)$$

in X_i,x,Q. The parameter $\theta \geq 0$ is an effective tax rate specified below. A positive parameter, $\theta>0$, indicates distortionary taxation. The objective function (1) features a monopolist charging country-specific prices. Excludability of the product's use is the basis for monopoly pricing. Without exercising market power the cost of quality cannot be covered. Differentiating between $P(X_i,Q)$ and $p(x,Q)$ assumes that price discrimination by country is a technologically feasible and profit-maximizing strategy. Google Ads has been mentioned as an example.

It is worth emphasizing that the quality Q is set by the monopolist and independent of the number of other market participants and their behavior. This seems to conflict with the view that Google is a platform. A platform is usually modelled as a two-sided market where the quality increases with the number of users on both sides. The network externality exerted by visiting a platform is only rudimentarily modelled by eq. (1). Still, it is captured in the following sense.

As a monopolist, the platform operator has an incentive to internalize any network externalities. In doing so, the operator will distinguish between two kinds. One is exercised by fee-based platform uses, the other by gratuitous ones. The former externality is not modeled in objective function (1) because Q is a choice variable and not a function depending on the quantities demanded for price. The externalities of free platform use, on the other hand, can be thought of as captured. This is so for the following reason. If visits to a platform depend only on its quality, and the quality in turn depends on the number of visitors and the platform operator's investments, the quality can be modeled as a function of investment costs alone. Solving this functional relation for costs yields the monopolist's cost function, $C(Q)$. Eq. (1) is therefore an appropriate modeling of Google Ads pricing problem if the assumption is correct that a firm's willingness to bid in Google Ads auction primarily depends on platform traffic from non-paying users and less on traffic from other paying users.

The solution of eq. (1) relies on the assumption that the market countries M_i $(i = 1,..,N)$ are modelled symmetrically and that they import equal quantities. Let $X \equiv X_i$ $(i = 1,..,N)$ maximize the monopolist's objective function. The first-order conditions (FOCs) can then be written as

$$P + XP_X = W, \;\; p + xp_x = \frac{W}{1-\theta}\,, \tag{2}$$

$$NXP_Q + (1-\theta)xp_Q = C_Q\,. \tag{3}$$

Subscripts used in connection with functions indicate (partial) derivatives. Conditions (2) refer to quantities and require marginal revenues to equal marginal costs in each country. Condition (3) refers to quality and it requires the sum of marginal revenues to equal the marginal cost of quality.

Let $X(\theta, N, W)$, $x(\theta, N, W)$, and $Q(\theta, N, W)$ denote a solution of the FOCs and assume that Q, X, x are converging for $N \to \infty$ to some positive values. Such limit behavior is assumed throughout the following analysis. When the number of countries is already large, an additional market country has a vanishing effect on optimal quantities and quality. Satiation in the demand of quality or sufficient inelasticity in its supply is able to ensure the convergence of Q to some finite value. If the price of quantity increases in quality, $P_Q, p_Q > 0$, and if marginal revenue from selling quantity does not decrease with quality, $p_Q + xp_{xQ} \geq 0$, the partial derivatives of Q and x with respect to θ are as follows:

$$\frac{\partial Q}{\partial \theta} < 0, \text{ and } \frac{\partial x}{\partial \theta} \leq 0 \text{ if } W = 0, \text{ whereas } \frac{\partial x}{\partial \theta} < 0 \text{ if } W > 0. \quad (4)$$

$$\frac{\partial Q}{\partial \theta} \to 0 \text{ and } \frac{\partial x}{\partial \theta} \to \frac{W}{(1-\theta)^2}\frac{1}{2p_x + xp_{xx}} < 0 \text{ for } N \to \infty. \quad (5)$$

The inequalities (4) and the limit properties (5) are proved in Appendix A by relying on the second-order conditions of the monopolist's maximization (1). An increase of distortionary profit taxation has a negative effect on quality Q which, however, vanishes for $N \to \infty$. P_Q equally vanishes for $N \to \infty$ as follows from eq. (3). The effect on quantity x is more differentiated. It is weakly negative and it vanishes for $N \to \infty$ if the use is non-rival, $W = 0$. By contrast, it is (strictly) negative if the use is rival, $W > 0$, and it remains so even for $N \to \infty$.

The focus of the analysis will be on M_0's optimal policy when quality is endogenous and N tends to infinity. In this case $\frac{1}{N}\frac{xp}{XP}$ converges to zero which is interpreted as a *vanishing market share* of M_0. For the sake of simplicity, we also speak of a *small country* if its market share is vanishing.

4. Firms' demand in M_0

The monopolist's revenues from M_0 are expenditures of firms domiciled in M_0. They would not be made if they were not productive. Let $f(g(Q)x, z)$ denote the production function of M_0. x and z are substitutable factors with returns to scale being non-increasing by assumption. The cost of z is normalized to be one. Quality, Q, is assumed to have an augmenting effect on quantity, x. The function $g(Q)$ is weakly increasing and concave.

In the Google example, the function f features the objective of advertising which is to evoke demand. The quantities x and z measure digital and non-digital units of advertising, respectively, while quality Q measures the accuracy of Google's algorithm in hitting the target group. An increase in Q has the effect that the productivity of digital advertising increases.

Let

$$c(p,Q) \equiv \min\{px + z \mid f(g(Q)x, z) = 1\} \tag{6}$$

denote the unit-cost function of the firms domiciling in M_0. At the minimum, the price of x is equal to the marginal rate of technical substitution, $p = gf_x/f_z$. (f_x denotes the derivative of f with respect to its first argument so that $\frac{d}{dx}f = gf_x$.) Partial derivatives of the unit-cost function are obtained by applying standard calculus techniques ("Shephard's Lemma"):

$$c_p = x(p,Q) > 0 \text{ and } c_Q = -pxg'/g < 0. \tag{7}$$

An increase in p raises the costs of firms in M_0 whereas an increase in quality, Q, reduces them. For later reference we note the following properties proven in Appendix B:

Remark 1: (i) $$\frac{d}{dQ}c(p(x,Q),Q) = x[p_Q - p\frac{g'}{g}] < 0, \tag{8}$$

(ii) $$\frac{p_Q}{p} = \frac{g'}{g}(1 + \frac{x}{p}p_x). \tag{9}$$

Property (i) says that quality has a decreasing effect on firms' cost. There is a direct partial effect and an indirect one working via a change in the price of x. The sign of the total derivative is determined by the direct effect. Property (ii) means that the direct quality effect on price is equal to the indirect quality effect exerted via quantity if quality has an augmenting effect on x and if cost is minimized. This is easily verified if z is kept constant. In this case, assuming $f(gx,z) = 1$ and $p = gf_x/f_z$ is equivalent to assuming $gx = const$ and $p/g = const$. The relative increase in g induced by a marginal increase of Q, $\frac{1}{g}\frac{dg}{dQ}$, must then be the same as $\frac{1}{p}\frac{dp}{dQ} = \frac{p_Q}{p} + \frac{p_x}{p}\frac{dx}{dQ}$. Taking account of $dx = -\frac{g'x}{g}dQ$ and rearranging terms yields eq. (9). In Appendix B it is shown that the simplifying assumption $z = const$ is not needed to prove property (ii). This property has a noteworthy implication. If $0 < W = (1-\theta)(p + xp_x)$ and $p_Q \to 0$, $g' \to 0$ must hold for $N \to \infty$. I.e. there must be satiation in the demand of quality.

5. Unilateral tax policy of M_0: The general case

As mentioned, there is not just one method that countries can use to tax a non-resident firm's profit at source. M_0 may charge a withholding tax, ω, on payments to H or limit the extent to which firms in M_0 are allowed to deduct such expenses. The latter would mean that only some fraction $1-\sigma$ of the payments for imports, $xp(x,Q)$, may be deducted from taxable profit earned by firms domiciling in M_0. From H's perspective, $\sigma xp(x,Q)$ is revenue earned and taxed in M_0 at rate t. If $t\sigma=\omega$, limiting deduction has the same effect as withholding. Both work like an import tax. In what follows, it is assumed that M_0 charges a withholding tax.

Initially, it will be assumed that H takes no specific measures to mitigate double taxation. Hence, $\theta(\omega)\equiv\frac{\omega}{1-T}$ where $\omega>0$ is set by M_0. The next derivations aim at determining the effect a marginal increase of ω has on the indirect utility, $V(q,I)$, of a representative consumer living in M_0. The effect is determined assuming that the tax planer of M_0 internalizes the optimal reaction of the monopolist, who in turn internalizes the changing demand behavior of firms in M_0. Let consumer demand, $d_0(q,I)=1$, be normalized to be one. Furthermore, let tax revenue be returned to consumers as lump sum income and let the consumer price be given by the firms' unit cost, $I\equiv\omega xp$, $q\equiv c(p(x,Q),Q)$ with $x=x(\omega)=x(\theta(\omega),N,W)$ and $Q=Q(\omega)=Q(\theta(\omega),N,W)$. By Roy's identity, $V_q=-d_0\cdot V_I=-V_I$. Hence,

$$0<\frac{d}{d\omega}V(c(p,Q),\omega xp) \iff 0<\frac{dy}{d\omega} \tag{10}$$

with $\frac{dy}{d\omega}\equiv\frac{d}{d\omega}[\omega xp(x,Q)]-\frac{d}{d\omega}c(p(x,Q),Q)$

$$=xp+\omega\frac{d}{d\omega}[xp]-c_p p_x\frac{dx}{d\omega}-\frac{dc}{dQ}\frac{dQ}{d\omega}]$$

$$=xp+\frac{\omega}{1-T}\frac{d}{d\theta}[xp]-\frac{1}{1-T}[xp_x\frac{\partial x}{\partial\theta}+\frac{dc}{dQ}\frac{\partial Q}{\partial\theta}]. \tag{11}$$

The derivative $\frac{dy}{d\omega}$ can be interpreted as the marginal efficiency of the withholding tax. According to eq. (11), the derivative can be written as the sum of four individual terms. The leading term captures marginal tax revenue when assuming that the tax base does not erode; its sign is positive. All the other terms tend to be negative and they are additionally grossed up by the factor $1-T$. The second term captures tax base erosion while the remaining two terms capture negative effects on firms' cost. This cost increases for two reasons. The monopolist's supply price increases (weakly) and the quality decreases. Both imply that the firms' unit cost increases

(Remark 1). Overall, the sign of $dy/d\omega$ is generally indeterminate. A bit more can be said when turning to another decomposition of effects.

Interpret $QE \equiv p\frac{g'}{g}\frac{\partial Q}{\partial \theta}$ as direct quality effect on cost and $PE \equiv (1-T)p - \frac{d}{d\theta}p(x,Q)$ as price effect, both evaluated at $\omega = 0 = \theta$. According to eq. (11),

$$\frac{d}{d\omega}y|_{\omega=0} > 0 \iff PE + QE > 0\ . \tag{12}$$

The direct quality effect is negative while the sign of the price effect is ambiguous. The sign of PE is only positive if the import price $(1-\omega)p(x(\omega), Q(\omega))$ decreases in ω at $\omega = 0$:

$$0 > \frac{d}{d\omega}\text{import price}\,|_{\omega=0} = -p + \frac{1}{1-T}\frac{dp}{d\theta}|_{\theta=0}$$

$$\iff 0 < (1-T)p - \frac{dp}{d\theta}|_{\theta=0} = PE\ . \tag{13}$$

An immediate implication of equations (12) and (13) is

Proposition 1: In general, the sign of marginal efficiency, $dy/d\omega$, is indeterminate at $\omega = 0$. It is positive if

(i) the price effect is positive – meaning that the import price decreases – and

(ii) the direct quality effect on cost is sufficiently small.

Proposition 1 raises the question as to which particular assumptions ensure a decreasing import price and a small direct quality effect on cost. In what follows, this question is answered by differentiating between $W > 0$ and $W = 0$. By giving different answers I highlight the relevance which rivalry in the use of imported goods and services has for M_0's optimal tax policy.

6. Unilateral tax policy of M_0: Exogenously fixed quality

The case of exogenously fixed quality is the one that has been analyzed by Brander et al. (1984). Proposition 2 restates their result for the present model. A critical variable is the price elasticity of demand, $\varepsilon \equiv \frac{p}{x}\frac{\partial x}{\partial p}$.

Proposition 2: ("Non-digital goods and services") Assume quality to be exogenously fixed and assume profit not to be taxed in H. If $W > 0$ and if ε is increasing in x, the efficiency in M_0 increases in ω at $\omega = 0$.

The proof is straightforward. The quality effect vanishes, $QE = 0$, by assumption. The price effect is positive, $PE > 0$, as $T = 0$ and as

$$0 < p - \frac{dp}{d\theta} = p - p_x \frac{\partial x}{\partial \theta} = p - p_x \frac{p + x p_x}{2p_x + x p_{xx}} \tag{14}$$

$$\Leftrightarrow \frac{d\varepsilon}{dx} = \frac{d}{dx}\left(\frac{p}{x}\frac{\partial x}{\partial p}\right) = \frac{d}{dx}\left(\frac{p}{x}\frac{1}{p_x}\right) > 0 \; .$$

The second equality in eq. (14) follows from eq. (2): $\frac{\partial x}{\partial \theta} = \frac{W/(1-\theta)^2}{2p_x + x p_{xx}} = \frac{p + x p_x}{2p_x + x p_{xx}}$ at $\theta = 0$. □

Proposition 2 is a qualification of the general statement that a small country does not benefit from taxing imports. In the present scenario, M_0 benefits because imports fetch a price which exceeds the marginal cost of quantity, W. If the price elasticity of demand is increasing in x, M_0 can use the tax on imports to extract monopoly rent (Brander et al., 1984). By contrast, if the supplier in H were to serve abroad at marginal (variable) cost, $p = W/(1-\theta)$, M_0 would not benefit from taxing imports. In this case, the price effect is non-positive, $PE = p - \frac{dp}{d\theta} = p\left[1 - \frac{1}{1-\theta}\right] = -p\frac{\theta}{1-\theta} \leq 0$.

For digital services, $W = 0$, and exogenous quality, Q, the price effect is necessarily positive. In fact, $PE = p > 0$. Average revenue before tax, p, does not react to a change of θ if $W = 0$. This is a trivial implication of eq. (2). The sign of $d\varepsilon/dx$ is therefore of no particular relevance.

Remark 2: If $W = 0$ and if quality is exogenously fixed, the efficiency in M_0 increases in ω.

7. Unilateral tax policy of M_0: Endogenous quality

The results of Section 6 do not readily apply to the case of endogenous quality. As revealed by eq. (12), the sign of marginal efficiency depends on the size of the negative direct quality effect, QE. More can only be said if M_0's market share is vanishing. With a vanishing market share the impact of M_0's policy on supplied quality vanishes, $\frac{\partial Q}{\partial \theta} \to 0$. By relying on equations (5) and (11) and by evaluating at $\omega = 0 = \theta$, marginal efficiency can be written as

$$\frac{d}{d\omega} y|_{\omega=0} \to xp - \frac{x}{1-T} p_x \frac{\partial x}{\partial \theta} = x[p - \frac{W}{1-T} \frac{p_x}{2p_x + xp_{xx}}] \text{ for } N \to \infty. \tag{15}$$

When inserting $T = 0$ and $W = p + xp_x$, the bracketed expression on the RHS is identical with the RHS of eq. (14) which has been shown to be positive if the price elasticity of demand, ε, is increasing in x. This proves

Proposition 3: ("Non-digital goods and services"): If $W > 0$, $T = 0$, if the market share of M_0 is sufficiently small, and if the price elasticity of demand, ε, is increasing in x, the efficiency in M_0 increases in ω at $\omega = 0$.

If $W = 0$, the RHS of eq. (15) equals marginal tax revenue, xp. In this case, M_0 is incentivized to set ω as high as possible subject to the constraint that the monopolist does not stop serving M_0. This level is reached if tax payments, $(\omega + T)xp$, equal revenues, xp, i.e. if $\omega = 1 - T$. In this case, we say that M_0 *skims off the profit contribution net of tax*.

Proposition 4: ("Digital services"): If $W = 0$ and if the market share of M_0 is sufficiently small, M_0 is incentivized to skim off the profit contribution net of tax.

Propositions 3 and 4 perfectly parallel the findings of Section 6. If quality is endogenous, one only has to add the assumption that the market share of M_0 is sufficiently small. If this is the case, the effect of M_0's tax policy on quality, $\partial Q/\partial\theta$, vanishes which comes close to assuming that quality is exogenously fixed. Propositions 3 and 4 are noteworthy for the following reasons. The first concerns the conception of smallness.

Usually, a country is called small if its policy has no effect on the terms of trade. In the present model, the terms of trade are given by the import price, $(1-\omega)p(x(\omega), Q(\omega))$. Proposition 3 only holds if the import price decreases in ω at $\omega = 0$. Therefore, M_0 cannot be considered being small in the usual sense. On the other hand, Propositions 3 and 4 only hold if the market share of M_0 is sufficiently small. It is in this sense that the propositions rely on an assumption of smallness.

Note that Proposition 4 is not just a special case of Proposition 3. This is so as Proposition 3 relies on assumptions obviously not needed to prove Proposition 4. More precisely, Proposition 4 makes no particular assumption on the elasticity of demand and it holds for arbitrary values of $T \in [0,1)$ and $\omega \in [0,1-T]$. In contrast, Proposition 3 does only hold at $T = 0 = \omega$. If the tax rate T is to be positive, one has to add other restrictive assumptions. In Appendix C this is shown for linear demand. In this case, T may not be larger than one half. Otherwise, examples are easily shown to exist where a marginal tax on the monopolized supply of non-digital goods and services has a decreasing effect on the efficiency in M_0. This is caused by an upward move of the import price, $(1-\omega)p$, which results if the direct effect of ω on the import price is dominated by the indirect one, $p < (1-\omega)\frac{dp}{d\omega} = \frac{1}{1-T}\frac{dp}{d\theta}$, when evaluated at $\omega = 0$. The larger T is, the more likely this is to happen. The difference in the assumptions of Propositions 3 and 4 corroborates the claim that the incentive to tax monopolized imports changes when $W = 0$ replaces $W > 0$.

8. Alternative pricing strategies

So far it has been assumed that the monopolist chooses a profit maximizing price for each market country separately. As mentioned there may be reasons why such price discrimination is neither feasible nor optimal. If the monopolist does not differentiate prices internationally, the attempt of M_0 to extract rent income is self-defeating. M_0 finds itself in the role of a small country, unable to change the terms of trade to its own advantage. The only effect of withholding and setting $\omega > 0$ is a loss of internal efficiency, $dy/d\omega < 0$.

A more interesting case is given when market countries join to pursue a rent-extracting tax policy. An openly agreed coordination is not needed; tacit collusion will do as well. To discuss this case formally, the roles of countries are slightly changed. More particularly, the monopolist is now assumed to serve its home market H in addition to the N foreign markets M_0 and all foreign market countries are modelled symmetrically. The variables X, P now denote price and quantity as related to H. The monopolist can then be assumed to maximize

$$\Pi \equiv X[P - W] + Nx[(1-\theta)p(x,Q) - W] - C(Q) \tag{16}$$

in X, x, and Q. FOCs are given by equations (2) and

$$XP_Q + N(1-\theta)xp_Q = C_Q \ . \tag{17}$$

Let $X(\theta, N, W), x(\theta, N, W)$, and $Q(\theta, N, W)$ denote a solution of the FOCs and assume that X, x, Q converge to some positive values for $N \to \infty$. In this case, the marginal price of quality earned on exports must be vanishing, $p_Q \to 0$. Because of eq. (9), $g'(p + xp_x)$ then vanishes as well. Either the optimal supply of quality approaches its level of satiation with $g' = 0$ or the supplied service is digital, $W = 0$.

For signing $dy/d\omega$ we return to eq. (11) and assume $W > 0$. The second bracketed expression on the RHS of eq. (11) then vanishes for $N \to \infty$. As a result, $dy/d\omega$, equals the sum of a positive revenue effect and an efficiency-reducing quantity effect. As shown in Appendix D, $\partial x/\partial\theta$ is a multiple of W and we obtain

$$\frac{d}{d\omega} y|_{N\to\infty} \ \to xp - \frac{1}{1-T}\frac{W}{(1-\theta)^2}\frac{xp_x - \omega W/(1-\theta)}{p_{QQ}(2p_x + xp_{xx}) - xp_{xQ}^2} p_{QQ}. \tag{18}$$

If $W = 0$, $\partial x/\partial\theta$ and $\partial Q/\partial\theta$ vanish for $N \to 0$. The efficiency-reducing quantity effect can then be ignored. In this case, M_0 is incentivized to skim off the profit contribution net of tax and to set $\omega = 1 - T$. The difference to Proposition 4 is that all market economies now do the same. As a result, the monopolist is unable to earn any profit from exporting the digital service.

Proposition 5: If the number of market economies grows to infinity, they import quality at a vanishing marginal price. The optimal tax on the import of non-digital goods and services trades off the revenue effect against an efficiency-reducing quantity effect. By contrast, digital services can be taxed without loss of internal efficiency. In their case, the market economies are incentivized to skim off the profit contributions net of tax.

9. Home's unilateral policy options

The analysis has shown that sufficiently small market countries have an incentive to tax monopolized imports even if the quality is endogenous. Smallness and country-specific pricing ensure that the negative effect on supplied quality is negligible. If there are many small market countries, the monopolist finds it optimal to charge a vanishing marginal price of quality on exports. If the marginal cost of quantity vanishes as well, market countries can tax the import at no cost of internal efficiency. The monopolist is then unable to earn any profit contribution on export. This will harm the monopolist's ability to cover the cost of developing quality. If Home is too small to generate the revenue needed to cover the cost of development, the digital service is not supplied at all.

The implications are less drastic in the case of non-digital goods and services. First, the monopolist is more likely to earn profit contributions from export because it may be too inefficient for market economies to skim off the full local monopoly rent. Secondly, Home could consider increasing the profit tax rate, T. As shown by eq. (18) this would raise the efficiency-reducing quantity effect. (Decreasing T would be counter-productive.) Thirdly, Home could resort to a policy of retaliation as discussed next.

If H equally taxes imports from M_0, M_0 risks being worse off than it would have been without taxing its own imports. In the old economy of non-digital goods and services, small countries even have the least incentive to engage in a war of tariffs. It has to be noted, however, that the prerequisite for retaliation is reciprocity in trade and that trade in digital services lacks reciprocity. The reason for the lack of reciprocity is technological. The production of digital services often has the characteristics of a natural monopoly. There are economies of scale and scope and often network externalities. In addition, spillover effects in R&D bring about regional concentration. The emergence of regionally concentrated natural monopolies even fosters growth from which the whole world benefits. It would only harm global efficiency if perfectly substitutable digital services were supplied by independent producers or if digital R&D were spread evenly throughout the world. Achieving balanced trade in digital services is therefore neither efficient nor would it be competitively sustainable. Countries importing digital services cannot and should not rely on the promise that they have a fair (and not tax driven) chance at some time in the future of switching to the role of a digital service exporter. Yet reciprocity in trade is a necessary condition for retaliation. Thus when a small country chooses to levy a tax on digital services, retaliation within the digital sector is unlikely be an effective threat.

10. International policy coordination

As before, the focus is on a monopolist wishing to export a digital service to many countries at country-specific prices. The monopolist is assumed to stop exporting if there are no positive profit contributions from abroad. Under such circumstances, non-cooperative policy is drastically inefficient calling for international policy coordination.

Let us assume that countries negotiate the allocation of the taxable profit earned in market economies and that the outcome of the negotiation can be modelled by Nash's bargaining solution. If agreement fails, Home's outcome is home profit, $XP - C$. The disagreement outcome of each of the symmetric market countries is zero. Let $1 - \nu \in (0,1)$ denote the bargaining power of Home and ν/N the one of a market country, M_0. Let σ be the share of profit, xp, taxed in M_0 and $1 - \sigma$ the share taxed in H. If the bargaining outcome is a parameter σ larger than zero but smaller than one, we speak of *profit splitting*. Double taxation is ruled out by construction. The parameter of split, σ, is determined by maximizing the Nash product, $[N(1-\sigma)xp]^{1-\nu} \cdot [\sigma xp]^{\nu}$. If xp were constant in σ, the trivial solution would be $\sigma = \nu$. The share of xp taxed by H would exactly reflect H's bargaining power. However, xp varies with σ. The FOC therefore is:

$$0 = (1-\nu)N\left[-xp + (1-\sigma)\frac{d}{d\sigma}(xp)\right][\sigma xp] + \nu N[(1-\sigma)xp][xp + \sigma\frac{d}{d\sigma}(xp)]$$

$$\Leftrightarrow [(1-\nu)\sigma - \nu(1-\sigma)]xp = \sigma(1-\sigma)\frac{d}{d\sigma}(xp) \qquad (19)$$

To determine the derivative in this equation, we have to maximize the monopolist's profit after tax, $(1-T)[PX - C] + N[1 - T(1-\sigma) - t\sigma]px$ in X, x, Q. The FOCs are eq. (2) with $W = 0$ and eq. (17) with $= \sigma\frac{t-T}{1-T}$. Taxation is distortionary if $\sigma > 0$ and $T \neq t$. The production of quality is effectively subsidized if $\sigma > 0$, $T > t$, and it is effectively taxed if $\sigma > 0$, $T < t$. Subsidization (taxation) results when the return on quality is taxed in foreign markets at a rate which is lower (higher) than the rate at which costs are offset in H.

By eq. (2), $\frac{d}{dx}(xp) = 0$ for $W = 0$. Hence, $\frac{d}{d\sigma}(xp) = xp_Q\frac{\partial Q}{\partial\sigma} = -xp_Q\frac{\partial Q}{\partial\theta}\frac{T-t}{1-T}$. The sign of $\frac{\partial Q}{\partial\theta}$ is already known to be negative for large values of N and p_Q tends to zero when $N \to \infty$. Hence,

$$\frac{d}{d\sigma}(xp) > 0 \Leftrightarrow T > t \text{ for large } N \text{ and } \frac{d}{d\sigma}(xp)|_{N\to\infty} \to 0 \,. \qquad (20)$$

A first implication of eq. (19) is that σ is neither zero nor one. I.e. profit splitting results from bargaining over σ. A second implication is the equivalence of the two inequalities, $1-\sigma < 1-\nu$ and $T > t$. This means that the share of xp allocated to H is lower than H's bargaining power if, and only if, the tax rate of Home exceeds the tax rate of a market country. I.e. a relatively large tax rate of H weakens H's bargaining power. Let us say that Home is a high-tax country if $T > t$.

Proposition 6: For large values of N, the share of xp taxed by Home is lower than Home's bargaining power if, and only if, Home is a high-tax country. For $N \to \infty$, Home's share, however, tends to the parameter of Home's bargaining power.

Profit splitting is appealing for further reasons. It cannot only be expected to result from bargaining over the apportionment of the taxable profit earned in market countries. It also secures inter-country tax equity, as argued by Richter (2019). Furthermore, it provides certain resilience against tax competition for the location of digital developments. This is easily shown by comparing the monopolist's aggregate tax payments when domiciling in H and serving H, M_0 with those the monopolist would have to pay if it relocated and served from M_0:

$$T[PX + (1-\sigma)px - C] + t\sigma px \;\le\; t[px + (1-\sigma)PX - C] + T\sigma PX$$

$$\Leftrightarrow \qquad (T-t)[PX + px - C] \;\le\; (T-t)\sigma[PX + px]$$

Assuming $T > t$, this inequality is equivalent to

$$(1-\sigma)[PX + px] \le C \;\Leftrightarrow\; \rho \equiv (PX + px - C)/C \le \sigma/(1-\sigma). \tag{21}$$

The interpretation is that a firm producing quality in a high-tax country and serving markets from there cannot save on tax payments by simply relocating to a low-tax country if the expected rate of return ρ does not exceed $\sigma/(1-\sigma)$. If $\sigma/(1-\sigma)$ were infinite, relocating from a high-tax to a low-tax country would never pay off. Therefore, a high-tax country with a strong digital sector will favor a large value for σ. There is, however, a contrary reason for H to favor a small value for σ, since any increase in σ decreases H's tax revenues. Hence, when negotiating the splitting parameter σ, a high-tax country with a strong digital sector faces a tradeoff. The national interest of low-tax countries with weak digital activity is diametrically opposed: A high level of σ is good for tax revenue whereas a low σ makes it more competitive for the location of those digital activities promising high rates of expected return.

Proposition 7: For high-tax countries, profit splitting is appealing because it provides certain resilience against tax competition for the location of digital developments.

The reasoning in favor of profit splitting is not readily applicable to non-digital goods and services. Although profit splitting provides certain resilience against tax competition even if non-digital goods and services are produced, H has the more direct option to retaliate. This threat is credible since, unlike trade in digital services, trade in non-digital goods and services tends to be reciprocal.

11. Concluding remarks

Quite a number of countries can be seen to be moving towards expanding source taxation of online business activities (Bunn et al., 2020). A topical example is the proposal of the European Commission (2018) to levy a DST. Its introduction has been justified by the Commission as a first step towards achieving “fair taxation of the digital economy”. None the less, the DST would violate current international tax standards and has been criticized by tax experts on these grounds.

The present paper interprets the taxation of digital services as the attempt to extract rent income earned on their monopolized import. Such an interpretation is substantiated by the study of the special case in which a monopolist supplies business services from Home to (foreign) market countries. Current tax standards assign the right to tax the profit earned on such remote supplies exclusively to Home. According to Brander and Spencer (1984) such circumstances are conducive to non-cooperative attacks. Market countries are incentivized to levy a tax on the import of the monopolized supplies. The present paper argues that this incentive is considerably strengthened if the import concerns digital services. If a marginal tax on the import of non-digital goods and services were to be an efficient policy, certain assumptions would have to be fulfilled. E.g., the price elasticity of demand would have to be increasing (Propositions 2 and 3). Furthermore, the importing country would have to take into account the political risk of retaliation. All this is different if digital services are imported. Their use is largely non-rival; the marginal cost of quantity is insignificant. This has policy implications. The importing country is incentivized to skim off the full local profit contribution (net of Home’s tax; Proposition 4). The assumption of an increasing price elasticity of demand is no longer needed. However, the monopolist must be assumed to set country-specific prices. Furthermore, the market share of the importing country must be sufficiently small so that the import tax has a vanishing effect on supplied quality. Sufficiently small countries even run a reduced risk of retaliation. Technology and language make small countries unlikely hosts of digital innovation. Retaliation would have to be targeted at a non-digital sector which could be considered a disproportionate action and initiate an escalating trade war.

The fact that small countries have an incentive to tax the import of digital services is worrying. If many countries seize the opportunity, it clearly harms digital innovation and reduces the variety and quality of digital services (Proposition 5). In this paper, it is argued that an appealing solution to this global policy dilemma is an internationally coordinated tax regime in which the profit earned on remote digital business services is split for the purpose of taxation. Such profit splitting can be expected to result when Home bargains with the market countries over the allocation of taxable profit (Proposition 6). High-tax countries with a strong digital sector even have reason to endorse profit splitting as it provides certain resilience against tax competition for the hosting of profitable developments (Proposition 7). A similarly simple, yet different rule of profit allocation is used by the Secretariat of the OECD (2020) to illustrate its proposed consensus solution to the tax challenges arising from digitalization. The Secretariat's proposal assumes that 20 percent of taxable residual profit ("Amount A") earned worldwide is allocated to market jurisdictions. By contrast, the analysis of the present paper suggests that profit splitting should be limited to those profit contributions that are earned in market countries. Although this specific form of profit splitting is not in line with current international tax law, it certainly deserves to be considered as a possible solution for the assignment of taxing rights in the digital economy.[4]

12. Appendices

A) The derivatives $\frac{\partial Y}{\partial N}$ and $\frac{\partial Y}{\partial \theta}$ are obtained for $Y = X, x, Q$ by solving the following system of equations:

$$\begin{bmatrix} 2P_X + XP_{XX} & 0 & P_Q + XP_{QX} \\ 0 & 2p_x + xp_{xx} & p_Q + xp_{xQ} \\ N(P_Q + XP_{QX}) & (1-\theta)(p_Q + xp_{xQ}) & NXP_{QQ} + (1-\theta)xp_{QQ} - C_{QQ} \end{bmatrix} \begin{bmatrix} \partial X \\ \partial x \\ \partial Q \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & \frac{w}{(1-\theta)^2} \\ -XP_Q & xp_Q \end{bmatrix} \begin{bmatrix} \partial N \\ \partial \theta \end{bmatrix} \tag{22}$$

[4] For an in-depth discussion of the pros and cons of profit splitting, see Richter (2019).

The determinant of the Hessian matrix is

$$D = [2p_x + xp_{xx}]D_{22} - (1-\theta)[p_Q + xp_{xQ}]^2[2P_X + XP_{XX}]\ . \qquad (23)$$

$D_{22} = [2P_X + XP_{XX}]\big[NXP_{QQ} + (1-\theta)xp_{QQ} - C_{QQ}\big] - N[P_Q + XP_{QX}]^2$ denotes the cofactor associated with the second element in the second column. The second-order conditions imply positivity of D_{22} and negativity of D, $2P_X + XP_{XX}$, and $2p_x + xp_{xx}$. By Cramer's rule,

$$\frac{\partial Q}{\partial N} = -XP_Q \frac{[2P_X+XP_{XX}][2p_x+xp_{xx}]}{D} > 0 \iff P_Q > 0\ , \qquad (24)$$

$$\frac{\partial Q}{\partial \theta} = -\frac{xp_Q}{XP_Q}\frac{\partial Q}{\partial N} - \frac{W}{1-\theta}\frac{[2P_X+XP_{XX}][p_Q+xp_{xQ}]}{D} < 0 \Longleftarrow p_Q > 0, p_Q + xp_{xQ} \geq 0\ , \qquad (25)$$

$$\frac{\partial x}{\partial N} = XP_Q \frac{[2P_X+XP_{XX}][p_Q+xp_{xQ}]}{D} \geq 0 \iff P_Q[p_Q + xp_{xQ}] \geq 0\ , \qquad (26)$$

$$\frac{\partial x}{\partial \theta} = -\frac{xp_Q}{XP_Q}\frac{\partial x}{\partial N} + \frac{W}{(1-\theta)^2}\frac{D_{22}}{D} \leq 0 \Longleftarrow p_Q[p_Q + xp_{xQ}] \geq 0$$

$$\text{and } \frac{\partial x}{\partial \theta} < 0 \text{ if additionally } W > 0\ . \qquad (27)$$

X, x, Q are converging for $N \to \infty$ by assumption. Therefore, the absolute values of D_{22} and D tend to infinity when $N \to \infty$. Hence,

$$\frac{\partial Q}{\partial N}, \frac{\partial x}{\partial N}, \frac{\partial Q}{\partial \theta} \to 0 \ \text{ and } \ \frac{\partial x}{\partial \theta} \to \frac{W}{(1-\theta)^2}\frac{1}{2p_x+xp_{xx}} < 0 \ \text{ for } \ N \to \infty. \ \square \qquad (28)$$

B) The partial derivatives p_Q and p_x are obtained by applying the implicit function theorem to the system of the two equations, $f(gx, z) = 1$ and $p = gf_x/f_z$, and by treating p, z as endogenous variables and Q, x as exogenous parameters. One obtains

$$\frac{\partial p}{\partial Q} = \frac{d}{dQ}\left(\frac{gf_x}{f_z}\right) - \frac{f_x}{f_z}g'x\frac{d}{dz}\left(g\frac{f_x}{f_z}\right) = pg'\left(\frac{1}{g} + x\Delta\right) \qquad (29)$$

with $\Delta \equiv \left(\frac{f_{xx}}{f_x} - \frac{f_{zx}}{f_z}\right) + \frac{f_x}{f_z}\left(\frac{f_{zz}}{f_z} - \frac{f_{xz}}{f_x}\right) < 0$. In the case of constant returns to scale such negativity follows from standard assumptions on partial derivatives of f. Negativity is equally obtained if returns to scale are decreasing, $gxf_x + zf_z = \lambda f$ with $\lambda < 1$. To see this one has to take partial derivatives of the latter identity with respect to x, z and to solve the resulting system of equations for $x, z > 0$. It then follows that the terms of Δ in round brackets must be negative. This implies that the sign of $\partial p/\partial Q$ is generally indeterminate. The sign becomes negative if $pg'/g > 0$ is subtracted from eq. (29):

$$\frac{d}{dQ}c(p(x,Q),Q) = x[p_Q - p\frac{g'}{g}] = pg'x^2\Delta \ < \ 0 \tag{30}$$

This proves property (i) of Remark 1. Similarly one proves

$$p_x = pg\Delta \ < \ 0. \tag{31}$$

Replacing $p\Delta$ in eq. (30) with p_x/p yields property (ii) of Remark 1.□

C) Set $p(x,Q) \equiv aQ^\alpha - bQx$ and assume $W > 0$. Solving the first-order condition (2) for x yields $x = \frac{1}{2bQ}[aQ^\alpha - W/(1-\theta)]$ and $p = \frac{1}{2}[aQ^\alpha + W/(1-\theta)]$.

$$\frac{d}{d\omega}y|_{\omega=0} \to x\left[p - \frac{W}{1-T}\frac{p_x}{2p_x + xp_{xx}}\right]$$

$$= \frac{1}{2}x\left[aQ^\alpha + W - \frac{W}{1-T}\right] = \frac{1}{2}x[aQ^\alpha - \frac{T}{1-T}W] \tag{32}$$

which is negative for sufficiently large $T > 1/2$. For $T \leq 1/2$, one obtains positivity $x > 0$ and therefore $aQ^\alpha - \frac{T}{1-T}W > \left[1 - \frac{T}{1-T}\right]W = \frac{1-2T}{1-T}W \geq 0$.□

D) The derivatives $\frac{\partial Y}{\partial \theta}$ are obtained for $Y = X, x, Q$ by solving the following system of equations:

$$\begin{bmatrix} 2P_X + XP_{XX} & 0 & P_Q + XP_{QX} \\ 0 & 2p_x + xp_{xx} & p_Q + xp_{xQ} \\ P_Q + XP_{QX} & N(1-\theta)(p_Q + xp_{xQ}) & XP_{QQ} + N(1-\theta)xp_{QQ} - C_{QQ} \end{bmatrix} \begin{bmatrix} \partial X/\partial\theta \\ \partial x/\partial\theta \\ \partial Q/\partial\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \frac{W}{(1-\theta)^2} \\ Nxp_Q \end{bmatrix} \tag{33}$$

Let D be the determinant of the Hessian matrix. Note that $p_Q \to 0$ for $N \to \infty$.

$$\frac{D}{N}|_{N\to\infty} \to (1-\theta)x[2P_X + XP_{XX}]\left[p_{QQ}(2p_x + xp_{xx}) - xp_{xQ}^2\right] < 0 \tag{34}$$

Assuming $p_Q + xp_{xQ} \geq 0$ implies $p_{xQ} \geq 0$ for $p_Q \to 0$. By Cramer's rule,

$$\frac{\partial Q}{\partial \theta}|_{N\to\infty} \to -\frac{W}{(1-\theta)^2}\frac{p_{xQ}}{p_{QQ}(2p_x+xp_{xx})-xp_{xQ}^2} = 0 \text{ for } W = 0 \text{ and } \leq 0 \text{ for } W > 0, \tag{35}$$

$$\frac{\partial x}{\partial \theta}|_{N\to\infty} \to \frac{W}{(1-\theta)^2}\frac{p_{QQ}}{p_{QQ}(2p_x+xp_{xx})-xp_{xQ}^2} = 0 \text{ for } W = 0 \text{ and } < 0 \text{ for } W > 0. \tag{36}$$

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